ARTICLES

Coarsening of a class of driven striped structures

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The coarsening process in a class of driven systems exhibiting striped structures is studied. The dynamics is governed by the motion of the driven interfaces between the stripes. When two interfaces meet they coalesce thus giving rise to a coarsening process in which $\ell(t)$, the average width of a stripe, grows with time. This is a generalization of the reaction-diffusion process $A+A\rightarrow A$ to the case of extended coalescing objects, namely, the interfaces. Scaling arguments which relate the coarsening process to the evolution of a single driven interface are given, yielding growth laws for $\ell(t)$, for both short and long times. We introduce a simple microscopic model for this process. Numerical simulations of the model confirm the scaling picture and growth laws. The results are compared to the case where the stripes are not driven and different growth laws arise.

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I. INTRODUCTION

Coarsening processes have been extensively studied both experimentally and theoretically over the past decades [1,2]. Most of these studies deal with the way a system approaches its *thermal equilibrium* state. For example, when a liquid is quenched to temperatures below the liquid-gas transition point a coarsening process takes place as the system evolves towards the equilibrium phase separated state. In these systems the order parameter (the density) is conserved and the evolution proceeds by either nucleation growth or by spinodal decomposition [3]. Similar processes take place in magnetic systems where a system evolves towards its magnetically ordered state when quenched below its critical point. In that case, however, the order parameter (the magnetization) need not be conserved by the dynamics and the details of the coarsening mechanism may differ from that of conserving dynamics.

Typically coarsening processes are characterized by a single growing length scale $\ell(t)$, for example the average domain size in the system. In many cases, at late times the system reaches a scaling regime where $\ell(t)\sim t^n$. The value of the exponent n usually depends on the symmetry of the system and its conservation laws. It has been found that for a scalar order parameter that is not conserved under the dynamics (as is the case in many magnetic systems) the growth exponent is $n=1/2$ [4]. This has been demonstrated theoretically by numerous studies of Ising-type models with Glauber dynamics [5]. On the other hand, when the order parameter is conserved (such as in liquid gas transitions and in phase separation in binary mixtures) the coarsening process is slower and the growth exponent was found to be $n=1/3$. This was first demonstrated by Lifshitz, Slyozov, and Wagner [6,7] and has been confirmed by many studies of Ising models with Kawasaki dynamics [8,2].

More recently, attention has focused on systems far from thermal equilibrium, driven by an external field [9]. In many cases such systems reach a steady state in which, unlike the equilibrium case, detailed balance is not obeyed. The lack of detailed balance allows for many phenomena which do not occur in thermal equilibrium, such as phase separation and symmetry breaking in one-dimensional systems [10,11]. Most of the recent studies of these systems have been focused on the properties of the steady state itself rather than on the evolution towards it. There are, however, indications that coarsening processes in these systems may be rather different from those of systems evolving toward their equilibrium state. For example, a study of the evolution of a driven Ising model with conserving dynamics has shown that in one dimension the average domain size grows as $t^{1/2}$ [12,13] rather than the usual $t^{1/3}$ expected for nondriven systems.

The presence of the drive introduces a preferred direction in space, making the systems inherently anisotropic. In many of these systems this results in striped structures. Typical examples are the stepped structures which occur in surface growth [14] and wind ripples formed in sand [15,16]. Many models of driven systems have been introduced and studied in recent years. The striped structures that naturally emerge may be oriented either parallel or perpendicular to the driving field, depending on the details of the dynamics [9]. For example, in a driven Ising model introduced by Katz *et al.* [17] stripes parallel to the driving field of alternating up and down spins are found at low temperatures. When the system is quenched to the low temperature phase at high values of the driving field narrow stripes are formed on a short time scale. On a longer time scale a coarsening process takes place in which the typical width of these stripes grows in time as the system evolves towards a fully phase separated state. This coarsening process is rather different from the one

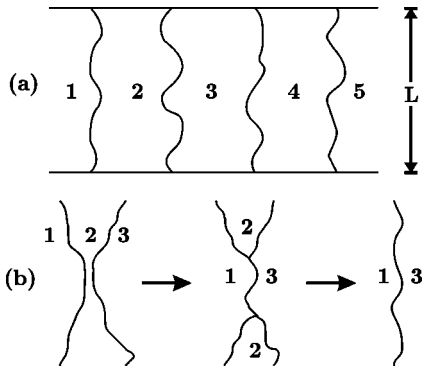


FIG. 1. Schematic representation of the model. (a) Domains are aligned parallel to each other, and are assigned one of $q\to\infty$ spin states in an ordered manner. (b) When two interfaces meet they coalesce into a single interface.

taking place in a system evolving towards its equilibrium state and is not well understood [18,19]. On the other hand, stripes perpendicular to the drive direction are observed in related spin-1 type models where two types of oppositely charged particles move in the presence of neutral vacancies [20–22,38].

Recently, a different class of driven models, where phase separation takes place even in one dimension, has been introduced [23–28]. In this class of models three or more types of particle are driven in a preferred direction under local dynamics. The important feature in these models is that the *local* dynamics results in a phase separated state even in one dimension ($D=1$). It has been demonstrated that in $D=1$ the coarsening process which accompanies the phase separation is slow, by which it is meant that the average domain size grows only logarithmically with time. An extension of one of the models belonging to this class to two and higher dimensions [29] showed that in dimensions greater than one stripes of alternating types of particles are formed perpendicular to the driving field. The width of the stripes along the direction of the driving field is found to grow as $\ln(t)$, which is the same growth law as the one dimensional case. This is related to the fact that the interfaces which separate adjacent stripes are macroscopically smooth.

Given the common occurrence of striped structures in different classes of driven models, it is of interest to explore the possible coarsening phenomena within a wider range of models exhibiting these structures. For example, in the preceding paragraph we described a driven system with conserving dynamics exhibiting slow coarsening. This behavior can be attributed to the smoothness of the interfaces separating neighboring stripes. A natural question is as to how the coarsening of stripes is altered when one considers more general scenarios such as, say, nonconserving dynamics or dynamics which leads to rough interfaces separating the stripes.

To investigate this issue we study in this paper a simple model for the evolution of driven striped structures. In the model stripes emerge oriented perpendicular to the driving field. The microscopic dynamics is nonconserving and rough interfaces between the stripes arise. We consider a system of infinite extent in the direction parallel to the drive and of finite size L in the other directions [see Fig. 1(a)]. A detailed definition of the microscopic model and its dynamics is given in Sec. II. Here we just describe the salient features.

(i) A microscopic state of the system is given by the configurations of the interfaces separating adjacent stripes [Fig. 1(a)]. These interfaces are assumed to be single valued, that is no bubbles or overhangs are present.

(ii) Each interface evolves under local driven dynamics. For example, one can consider growth dynamics belonging to the KPZ universality class. All interfaces evolve under the same dynamical rules, and thus, in particular, they all move in the same direction and with the same average velocity.

(iii) When two interfaces meet at a point, they locally merge to form a single interface. The evolution is such that after some time the entire two interfaces coalesce [Fig. 1(b)].

As a result of this dynamical process the number of interfaces keeps decreasing and the average width of a stripe $\ell(t)$ increases with time. One is interested in studying the details of this coarsening process.

One can think of the model in analogy to a $q\to\infty$ state Potts system, where each site is assigned a spin variable $S=1,2,\ldots,q$. In the initial configuration the spins in each stripe are given the same state and the state associated with each stripe increases in an ordered sequence 1,2,3, . . . [see Fig. 1(a)]. Each state $S=i$ propagates into states $S=j$ where $j>i$. Clearly, the dynamics is such that the order parameter (the density of particles of type S) is not conserved since the number of particles of a given state changes in time. Related models have previously been considered in the context of cyclic food chain [30] and the evolution of the spatial mosaic of single-species domains was studied.

As a general motivation for the type of model we study, one may think of the spin state as representing the height of a D-dimensional terraced surface. Thus the striped structure corresponds to a sequence of terraces of increasing height; an interface between two stripes corresponds to a step. The surface evolves through particles being adsorbed or evaporated from the steps. In this picture two coalescing interfaces correspond to a terrace of a given height being eliminated from the system. However, the model to be studied in this paper should not be viewed as a microscopic description of this particular growth process. Rather it provides a very simple dynamics that leads to coarsening of stripes with no bubbles or overhangs in the interfaces between stripes. In order to construct a realistic microscopic model for a particular system (such as the terraced surface) one would have to add other features that would complicate analysis of the model.

The dynamical process described above is a generalization of the reaction diffusion process $A+A\to A$, in which diffusing A particles (either with or without a drive) undergo a merging reaction as they collide. In the present generalization the coalescing objects are not particles but extended objects, i.e., the interfaces between stripes, which are manifolds in $d=D-1$ dimensions. The $A+A\to A$ reaction diffusion process has been studied in detail over the years [31]. It has been found that for both biased and unbiased diffusion the density (or equivalently the average distance between particles) decays as $t^{-D/2}$ for $D<2$ and t^{-1} for $D>2$ [32]. It is of interest to investigate how this behavior is changed for the

generalization to coalescing manifolds. In the present contextthe manifolds themselves evolve in time, due to a roughening process, which could also affect the behavior.

The scaling properties of driven manifolds are different from those of stationary ones. It is of interest to consider the evolution of striped structures when the interfaces are nondriven. In this case the dynamics can be related to an energy function. As we shall see, the coarsening process is affected by whether or not the interfaces are driven, unlike the case of coalescing particles described in the previous paragraph.

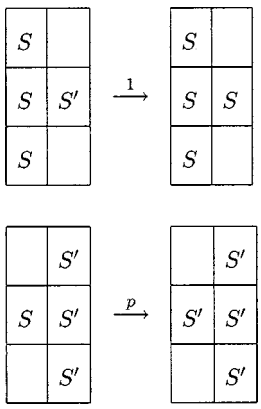
In studying the model we find that the coarsening process may be well understood by considering the scaling properties of an isolated interface. We find that the evolution is different at early and late times. In both regimes the average domain size $\ell(t)$ grows algebraically in time. The different regimes, however, are characterized by different exponents. These exponents are determined by the universal properties of the isolated interfaces and can therefore be deduced using scaling arguments. Moreover, the exponents depend on whether the interfaces are driven or not.

The paper is organized as follows. In Sec. II we define the model. In Sec. III a scaling argument which is based on the properties of an isolated interface is used to deduce the coarsening behavior. Monte Carlo simulations which verify the scaling argument are described in Sec. IV. In Sec. V the special case where the interfaces are nondriven and energy function exists is studied using similar scaling arguments and Monte Carlo simulations. We conclude in Sec. VI with a discussion.

II. THE COALESCING INTERFACES MODEL

Our model is defined on a D-dimensional hypercubic lattice with periodic boundary conditions. The lattice is infinite along one of its axes, which we label x, and is of size L along the d other axes labeled $\mathbf{y}=(y_1,y_2,\ldots,y_d)$. Each site $\mathbf{r}=(x,\mathbf{y})$ can be occupied by one of $q\to\infty$ species of particles $S_{\mathbf{r}}=1,2,\ldots,q$. In the initial configuration particles are arranged in stripes of single species perpendicular to the x direction. The stripes are in increasing order of particle type, their average extent along the x direction is ℓ_0 and the interfaces between them are flat. We are interested in constructing a model where species i propagates into species j when $j>i$. In view of the initial configuration this can be simply achieved through the dynamics we now describe.

The model evolves under random sequential dynamics according to the following rules: at each time step two neighboring sites along the x direction $\mathbf{r}=(x,\mathbf{y}),\mathbf{r}'=(x+1,\mathbf{y})$ are chosen randomly. If the particles at sites $\mathbf{r},\mathbf{r}'$ are not of the same species, then we proceed in one of the following ways: with probability $1/(1+p)$, where $p<1$, the update $S_{\mathbf{r}'}\to S_{\mathbf{r}}$ is made, but only if the particles at $\mathbf{r}$ and its neighbors along the $\mathbf{y}$ directions are of the same species; with probability $p/(1+p)$ the update $S_{\mathbf{r}}\to S_{\mathbf{r}'}$ is made, but only if the particles at $\mathbf{r}'$ and its neighbors along the $\mathbf{y}$ directions are of the same species. For example, in $D=2$ the net effect of this algorithm is for the following moves to occur with the relative rates indicated above the arrows:



(1)

where an empty square indicates that the move is attempted irrespective of the type of particle at that site.

Given the initial conditions, this dynamics ensures that the species index always increases to the right, along the x direction. This is equivalent to stating that the interfaces are single valued and that the order among the interfaces is preserved. Furthermore, when $p<1$ the dynamics is such that species i is driven preferentially to the right into neighboring species j where $j>i$. The case $p=1$ is special in that interfaces are not driven. In the following we consider $p<1$ and we discuss the special case $p=1$ towards the end of the paper. Finally note that in this dynamics the densities of the various species are clearly not conserved.

Under this dynamics an isolated interface, evolves as in a restricted solid on solid (RSOS) model, by which it is meant that changes in the x position of the interface as one moves in $\mathbf{y}$ directions are at most of magnitude 1. Moving interfaces of this kind belong to the Kardar-Parisi-Zhang (KPZ) universality class.

The coarsening process for a system of many interfaces can be understood in terms of the scaling properties of an isolated interface. These properties will be discussed in detail in the following section. However, at this point it is useful to outline the general picture that emerges. Starting from a dense collection of flat interfaces, at early times interfaces meet and coalesce due to the evolution of their width W, which measures the lateral extent of the interface. Thus ℓ, the average distance between neighboring interfaces, scales with the width of an individual interface. So for early times, the dependence of ℓ on time is governed by the time evolution of the width. In this regime the growth is independent of L and is determined by an exponent β

$$\ell(t)\sim W\sim t^{\beta} \quad \text{at early times.} \tag{2}$$

Since we are considering systems of finite size L in the directions transverse to the drive, the width saturates to some final value $W_{\mathrm{sat}}(L)$. Therefore at late times, when the width of the interfaces has already saturated, interfaces meet due to fluctuations, Δh, in their center of mass position $h(t)$. That is,

$$\ell(t)\sim\Delta h\sim\frac{t^{\gamma}}{L^{\phi}} \quad \text{at late times,} \tag{3}$$

where γ and ϕ are exponents to be determined. Notice the algebraic evolution of ℓ in the two regimes which contrasts with the logarithmic evolution of ℓ found in the model studied in Ref. [29].

The proposed picture is valid only at time scales much larger than that required for the process of coalescence of two interfaces. We find that at short times, the coalescence process takes in a finite time independent of L. This is because neighboring interfaces touch at a finite density of points. On the other hand, in the long time regime the time scale for coalescence is found numerically to be proportional to L. This has to be borne in mind when testing the above scaling picture.

The exponents β, γ, and ϕ are determined by the single interface behavior. In the next section we introduce the scaling analysis in a more quantitative way to obtain the exponents β, γ, and ϕ. Specifically, for $p<1$ the interfaces we consider belong to the KPZ universality class, so the two exponents are given in terms of known KPZ exponents. At the special point $p=1$ the interfaces belong to the Edwards-Wilkinson (EW) universality class, and the two exponents are changed accordingly.

III. SCALING ANALYSIS

As argued in the preceding section the evolution of the system is governed by the width of an isolated interface at short times. On the other hand, at long times the width saturates due to the finite lateral extent of the system, whereas the fluctuation of the center of mass (average x position) keeps increasing. Thus at long times the center of mass fluctuations govern the coarsening behavior. We now define these quantities more precisely and quantify their scaling behavior.

The width of an interface W is defined by

$$W^2 = \left\langle \frac{1}{L^d} \sum_{\mathbf{y}} (x_{\mathbf{y}} - h)^2 \right\rangle, \tag{4}$$

where $x_{\mathbf{y}}$ is the location of the interface along the x direction and h is the center of mass (average location) of the interface $h = \Sigma_{\mathbf{y}} x_{\mathbf{y}} / L^d$. The angular brackets denote an average over the dynamics, starting from the given initial condition. The fluctuation in the average location of the interface Δh is defined through

$$\Delta h^2 = \langle (h - \langle h \rangle)^2 \rangle, \tag{5}$$

where as before angular brackets denote an average over the dynamics.

In general, for a fluctuating interface the scaling behavior of the width is given by (see, for example, Ref. [34])

$$W = t^{\beta} f\left(\frac{t}{L^z} \right), \tag{6}$$

where z is the dynamic exponent. The scaling function $f(x)$ is constant for $x \ll 1$ while $f(x) \sim x^{-\beta}$ for $x \gg 1$. This implies

$$W \sim \begin{cases} t^{\beta}, & \text{for} \quad t/L^z \ll 1, \\ L^{\alpha}, & \text{for} \quad t/L^z \gg 1, \end{cases} \tag{7}$$

where $\alpha = \beta z$.

The fluctuation in the average location of the interface Δh is dimensionally equivalent to the width so we may write, in a fashion similar to Eq. (6),

$$\Delta h = t^{\beta} g\left(\frac{t}{L^z} \right). \tag{8}$$

For long times we expect the center of mass of the interface to diffuse, namely, we expect $\Delta h \sim t^{1/2}$, so that $g(x) \sim x^{1/2-\beta}$ for $x \gg 1$. Thus

$$\text{when} \quad t \gg L^z, \quad \Delta h \sim \frac{t^{1/2}}{L^{\varphi}}, \tag{9}$$

where $\varphi = z/2 - \alpha$.

For short times one needs to consider the specific universality class to which the interfaces belong. We start by restricting ourselves to the case $p<1$ (the case of $p=1$ will be discussed in Sec. V). In this case the dynamics is biased along the x direction, and the interfaces clearly belong to the KPZ universality class.

For KPZ interfaces at short times $t \ll L^z$ it has been argued [33] that there are two regimes depending on the ratio $(d+4)/z$. That is,

$$\text{when} \quad t \ll L^z, \quad \Delta h \sim \begin{cases} \dfrac{t^{\theta}}{L^{d/2}} & \text{for} \quad d+4 < 4z, \\ \dfrac{t}{L^{2(z-1)}} & \text{for} \quad d+4 > 4z, \end{cases} \tag{10}$$

where $\theta = (d+4)/2z - 1$.

We now use the width of the interface W and the fluctuation in its average position Δh to deduce the average distance between the interfaces as a function of time $\ell(t)$. As stated above, we will show that W controls the early time behavior while Δh controls the late time behavior of the system. We assume that initially the average distance between the interfaces ℓ_0 is much smaller than the width of a fully developed isolated interface L^{α}. It will be shown that two distinct coarsening behaviors are expected for $t \ll L^z$ and $t \gg L^z$.

Consider first the long time ($t \gg L^z$) behavior of the system. From Eqs. (7) and (10) one can see that $\Delta h \gg W$ and therefore Δh is expected to control the time scale on which interfaces coalesce. This implies that the average distance between interfaces should behave as

$$\ell \sim \frac{t^{1/2}}{L^{\varphi}} \quad t \gg L^z. \tag{11}$$

Next, consider the short time behavior ($t \ll L^z$). In this case it is straightforward to verify, using the hyperscaling relation

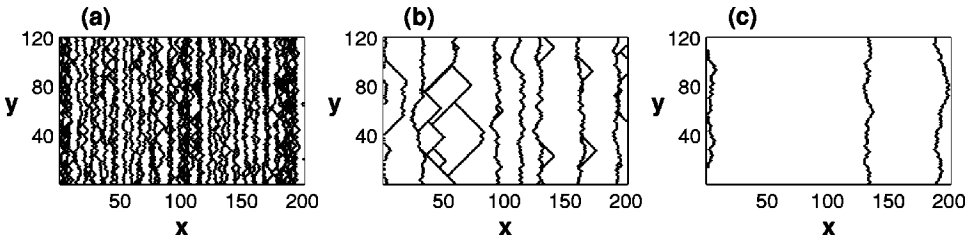


FIG. 2. Snapshots of a $d=1$ system at (a) $t=10$, (b) $t=10^3$, and (c) $t=10^5$. Here $L_x=1000$, $L_y=120$ and the average initial spacing between the interfaces is $\ell_0=4/3$ lattice sites. For clarity, only one quarter of the system in the $\hat{x}$ direction is presented.

$\alpha+z=2$, that $W \gg \Delta h$ for both regimes in Eq. (10). Thus at short times the coarsening behavior is controlled by the width of the interface and is expected to behave as

$$\ell \sim t^{\beta} \qquad t \ll L^z. \tag{12}$$

We have thus shown that Δh controls the long time behavior and W the short time behavior. The short time behavior is expected to be lost if the initial spacing between the interfaces is larger than $W_{\mathrm{sat}} \sim L^{\alpha}$. Also, in models for which the interfaces are smooth ($\alpha \leqslant 0$) the short time behavior is not expected to be seen. In the following we verify the predictions of the scaling analysis by numerical studies of the models in $D=2$ and $D=3$ dimensions.

IV. MONTE CARLO SIMULATIONS

We first consider the model in $D=2$ dimensions. The model is simulated on a lattice of size $L_x \times L_y$ where L_y gives the lateral size denoted by L in the previous sections. Periodic boundary conditions are used both in the x and y directions. Particles of different types are initially ordered in stripes parallel to the y axis. The positions of the interfaces between the stripes are chosen randomly, such that the mean distance between them is ℓ_0. At each step two neighboring sites in the x direction, which are occupied by different species are chosen randomly, and a move is made according to Eq. (1), where we have used $p=0$ in our simulations. In this way we maximize the speed of the simulation. After each such move, time is advanced by $\tau=1/\epsilon_x$, ϵ_x being the total number of nearest neighbor pairs in the x direction occupied by different species. The algorithm would be equivalent to a usual Monte Carlo simulation if τ were drawn from a Poisson distribution with mean $1/\epsilon_x$. Here we make the approximation $\tau=1/\epsilon_x$ which is valid as long as ϵ_x is large.

In Fig. 2 typical snapshots of the system at early, intermediate and late times are presented for a system of size $L_y=120$ and $L_x=2000$. The initial spacing between the interfaces is $\ell_0=4/3$. In Fig. 2(a) the early time behavior is shown. Here interfaces meet each other at several points indicating that their width is of the same order as the spacing between them. Note that in order to see such configurations the initial spacing between the interfaces must be much smaller than the final width W_{sat} of an isolated interface. At the short time regime the mean domain size ℓ is expected to scale according to Eq. (12) which for $d=1$ reads $\ell \sim t^{1/3}$.

At late times [Fig. 2(c)] the distance between interfaces is much larger than their width, and interfaces meet due to fluctuations in their center of mass location. In this regime ℓ is expected to scale according to Eq. (11), which in the $d=1$ case is just $\ell \sim t^{1/2}/L^{1/4}$.

In the intermediate regime Δh is comparable to W and we expect a crossover from the early to late time behavior. Figure 2(b) shows a typical configuration at this stage of the dynamics. We see from the figure that in this regime there is a significant probability of three interfaces meeting at a point. This results in some arrested local configurations where an interface is temporarily frozen in locally triangular forms and the apparent width of the interface becomes large. These configurations are then released when a fourth interface approaches from the left and sweeps through the triangular forms. This effect is a direct consequence of the RSOS condition of the model. We have explored variations of the model in which the RSOS condition is relaxed, and found a variety of model specific effects in this intermediate regime. Nevertheless the early and late time behaviors are not affected by the details of the model.

The overall picture we have outlined is quantified in Fig. 3. In this figure the average distance between stripes $\ell(t)$ is plotted as a function of time and is compared with the width W and fluctuation in the average interface location Δh of a single isolated interface. We calculate the average distance between the stripes through $\ell = L_x L_y / \epsilon_x$, where ϵ_x is de-

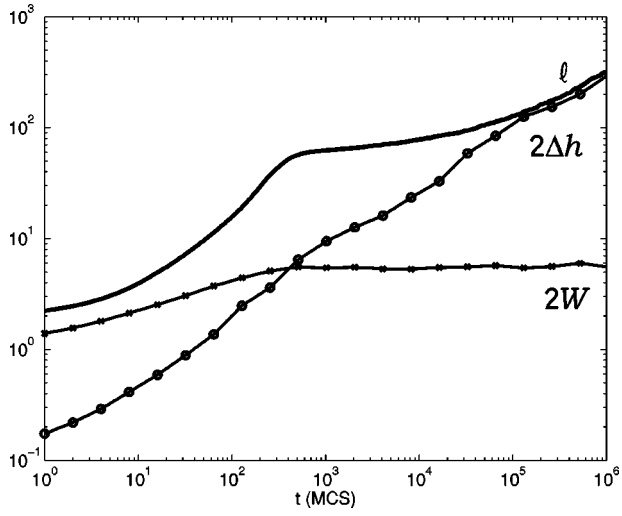


FIG. 3. Results of simulations for $D=2$. The mean spacing between interfaces ℓ as a function of time is shown, along with twice the width W and twice Δh, the fluctuations in average location of a single isolated interface. Note that at late times ℓ coincides with $2\Delta h$ as expected, while at early times ℓ is parallel to W. Here $L_x=1000$, $L=120$, and time is measured in Monte Carlo sweeps. The results are obtained from an average over 20 simulations for ℓ and 200 simulations for W and Δh.

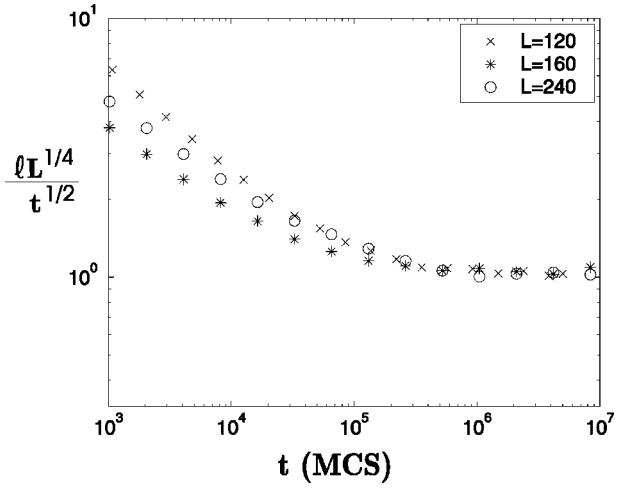


FIG. 4. Scaled average domain size $\ell L^{1/4}/t^{1/2}$ for systems of size $L=120$, 160, 240. The collapse at long times demonstrates the scaling predicted by Eq. 11.

fined as above, namely, the total length in the y direction of the interfaces. Similar results were obtained for various system sizes.

In Fig. 3 one can see that at early times the behavior of ℓ indeed closely follows that of $2W$, while at late times it closely follows that of $2\Delta h$. The factors of 2 are because the growth of ℓ is actually controlled by the sum of the fluctuations of two neighboring interfaces. In the intermediate regime where Δh and W are comparable ℓ deviates from both curves. This reflects the triangular configurations observed in this regime as discussed before.

We now make more precise tests of the scaling predictions. At late times ($t \gg L^z$) the scaling argument predicts a specific dependence of ℓ on t and L, $\ell \sim t^{1/2}/L^{1/4}$. To test this, in Fig. 4 we plot $\ell L^{1/4}/t^{1/2}$ against t for various system sizes. One can see that at late times the data indeed collapse, confirming the predicted behavior.

To get a more quantitative test of the predictions of the scaling argument at early times ($t \ll L^z$) is more difficult. The

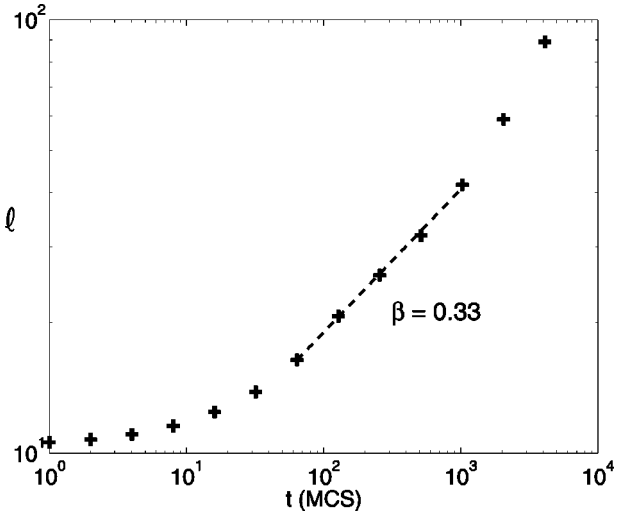


FIG. 5. Average domain size ℓ as a function of time, for a $D=2$ system with $L=10^4$. At short time one can see $\ell \sim t^{\beta}$ with $\beta = 0.33 \pm 0.01$ (dotted line).

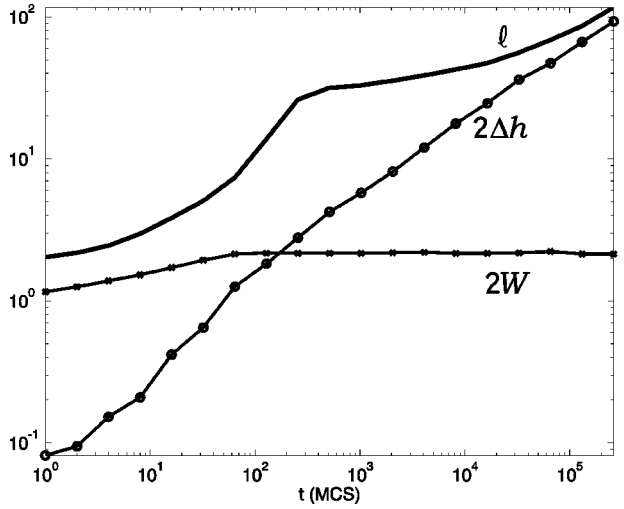


FIG. 6. Results of simulations for $D=3$. The mean spacing between interfaces ℓ as a function of time is shown, along with the width W and the fluctuations in average location Δh of a single isolated interface. Here $L_x=1000$, $L=L_y=L_z=20$, and time is measured in Monte Carlo sweeps. The results for ℓ are obtained from a single run, while those for W and Δh are averaged over 100 runs.

reason is that one has to access a regime in which $\Delta h \ll W$ while, at the same time, W displays its early time growth behavior. To obtain a small Δh, and to increase the time window in which W keeps growing, we require a large lateral size L. Also note that at short times a coalescence process of two interfaces occurs in a finite time which does not grow with L. This is because in this regime the neighboring interfaces touch at a finite density of points.

In Fig. 5 the average distance between the domains ℓ is plotted as a function of t for a system with $L=10^4$ and $\ell_0 = 10$. After a transient time the domain size $\ell \sim t^{\beta}$ over more than a decade, where $\beta = 0.33 \pm 0.01$ as expected.

Next we present results obtained from simulation of a three-dimensional system ($D=3$). In Fig. 6 the average distance between interfaces ℓ is plotted along with W and Δh for a system with $L_y=L_z=20$. Again one can see that at early times ℓ follows closely W while at late times it follows Δh. While these simulations confirm the general picture outlined above, quantitative estimates of the exponents would require a more elaborate study.

V. UNBIASED DYNAMICS

We now study the case $p=1$. In this case one can associate with the dynamics a well-defined local energy function. Since at each update the number of x neighbor pairs of different species is reduced, the energy is Potts-like in the x direction with unlike neighbor pairs costing energy. Unlike pairs in the y direction do not cost energy provided that the RSOS condition is satisfied. Any step of the dynamics carried out serves to lower the energy. In this way the dynamics can be thought of as a zero temperature system approaching an equilibrium state.

Single interfaces in this special case belong to the EW universality class, rather than to the KPZ class. Therefore the scaling analysis of Sec. III has to be applied to EW-type

interfaces. This modifies the values of the exponents and also the short time behavior of Δh.

For interfaces belonging to the EW universality class $\Delta h \sim t^{1/2}$ for all times. Therefore the scaling form (8) implies

$$\Delta h \sim \frac{t^{1/2}}{L^{\phi}} \tag{13}$$

for all times, with $\phi = z/2 - \alpha$ as before. We use the exact values of the EW exponents: $z=2$, $\alpha = 1 - d/2$, $\beta = 1/2 - d/4$ to find

$$\Delta h \sim \frac{t^{1/2}}{L^{d/2}}. \tag{14}$$

As in the previous case one can now determine the behavior of $\ell(t)$. For long times ($t \gg L^2$) using Eqs. (7), (13) and the values of the EW exponents one finds that $\Delta h \gg W$ and Δh is expected to control the mean distance between interfaces. Therefore

$$\ell \sim \frac{t^{1/2}}{L^{d/2}}, \quad t \gg L^2. \tag{15}$$

For short times ($t \ll L^2$) we have $W \sim t^{\beta} \gg \Delta h$. Therefore the coarsening is governed by the width of the interfaces and

$$\ell \sim t^{1/2 - d/4}, \quad t \ll L^2. \tag{16}$$

To summarize, in this case, just as in the $p \neq 1$ case, Δh controls the long time behavior and W the short time behavior. The difference between the two cases is due to the different values of the exponents α, β, and z and is thus a direct consequence of the universality class.

Monte Carlo simulations in $D=2$ with no bias support these findings. The simulations show that indeed at early times $\ell \sim t^{1/4}$ and at late times $\ell \sim t^{1/2}/L^{1/2}$ in agreement with the scaling argument and the known EW exponents. The results for the evolution of ℓ are qualitatively similar to those of the KPZ case and we do not display them here.

VI. DISCUSSION

In this work we have considered the evolution of a class of striped structures. A coarsening process takes place in which interfaces separating the stripes meet and coalesce. The dynamics may thus be viewed as a generalization of the particle reaction diffusion process $A + A \rightarrow A$ to extended objects, the interfaces. In this generalization the coalescing objects have internal structure that in turn affects the evolution. Thus one may have different scaling laws depending on the properties of the coalescing objects.

We studied a microscopic model defined through the dynamics of the interfaces that could realize both driven and undriven interfaces. Both cases lead to rough interfaces along with the average stripe width growing as a power law in time. However, the two cases lead to different scaling exponents. In the driven case an isolated interface exhibits KPZ behavior, while in the undriven case EW behavior is obtained. It is this scaling behavior of an isolated interface that determines both short and long time coarsening regimes of the stripes. We have shown that at early time the coarsening dynamics is determined by the width of the interfaces, while at short time it is determined by the fluctuations in the locations of the interfaces. The behavior is different from that of other driven models where the interfaces are smooth and the coarsening is logarithmically slow in time [29].

The analysis performed in this paper should apply also to other classes of interfaces. For example, the scaling behavior of moving fronts or interfaces has been studied for the case of ''pulled'' fronts, moving into an unstable phase. Recently, it has been suggested that such an interface is not of the KPZ type [35–37]. This indicates that there might be other possible types of scaling behavior than those we have studied in this paper. It would be of interest to explore such possible dynamics and study the resulting coarsening.

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